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Theoretical status of CP violation in Kaon decays $[\epsilon'/\epsilon]^\star$

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Abstract. This talk is devoted to review the status of theoretical calculations of ϵ'/ϵ. The focus is mainly on recent developments in non-lattice approaches to hadronic matrix elements.

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1 Introduction

CP violation in the neutral kaon system is characterized by the following ratios

$$\eta_{+-} \equiv \frac{A(K_L \to \pi^+\pi^-)}{A(K_S \to \pi^+\pi^-)} \simeq \epsilon + \epsilon' \tag{1.1}$$

$$\eta_{00} \equiv \frac{A(K_L \to \pi^0\pi^0)}{A(K_S \to \pi^0\pi^0)} \simeq \epsilon - 2\,\epsilon' \ . \tag{1.2}$$

While ϵ is a measure of CP violation in $K-\bar{K}$ oscillations, ϵ' describes CP violation in $|\Delta S|=1$ transitions. Both ϵ and ϵ' are now known to be non-zero: the PDG quotes [1] $|\epsilon| = (2.282 \pm 0.017) \times 10^{-3}, \Phi_\epsilon = (43 \pm 0.5)^o$, while the world average for ϵ'/ϵ is [2]

$$\mathrm{Re}\Big(\frac{\epsilon'}{\epsilon}\Big) = \frac{1}{6}\left(1 - \Big|\frac{\eta_{00}}{\eta_{+-}}\Big|^2\right) = (16.7 \pm 1.6) \times 10^{-4} \tag{1.3}$$

On the theoretical side ϵ'/ϵ is expressed as follows

$$\frac{\epsilon'}{\epsilon} = \frac{-ie^{i(\chi_2-\chi_0-\Phi_\epsilon)}}{\sqrt{2}|\epsilon|}\ \frac{\mathrm{Re}A_2}{\mathrm{Re}A_0}\ \left[\frac{\mathrm{Im}A_0}{\mathrm{Re}A_0} - \frac{\mathrm{Im}A_2}{\mathrm{Re}A_2}\right] \tag{1.4}$$

in terms of isospin amplitudes A_0 and A_2 (describing K^0 transitions to two-pion states with $J=0$, $CP=+1$ and isospin $I=0$ or $I=2$) and strong phases $\chi_{0,2}$, whose definition is given by:

$$A[K^0 \to \pi^+\pi^-] = A_0\, e^{i\chi_0} \ + \ \frac{1}{\sqrt{2}}\, A_2\, e^{i\chi_2} \tag{1.5}$$

$$A[K^0 \to \pi^0\pi^0] = A_0\, e^{i\chi_0} \ - \ \sqrt{2}\, A_2\, e^{i\chi_2} \ . \tag{1.6}$$

The phenomenological finding that $\frac{\mathrm{Re}A_2}{\mathrm{Re}A_0} \sim \frac{1}{22}$ ($\Delta I = \frac{1}{2}$ rule) implies a dynamical suppression of ϵ', while $\chi_0 - \chi_2 \sim 50^o$ implies that ϵ'/ϵ has a small imaginary part.

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The calculation of nonleptonic kaon amplitudes requires control not only over the underlying weak process at short distance but also on the effects induced by strong interactions from short- to long-distances, where non-perturbative effects became important. By integrating out heavy degrees of freedom in the underlying theory one derives the effective strangeness-changing hamiltonian at the renormalization scale μ:

$$\mathcal{H}_{\mathrm{eff}}^{\Delta S=1} = \frac{G_F}{\sqrt{2}}\, V_{ud}V_{us}^*\ \sum_i\ C_i(\mu)\, Q_i(\mu) \tag{1.7}$$

Wilson coefficients $C_i(\mu) = z_i(\mu) - y_i(\mu)(V_{td}V_{ts}^*)/(V_{ud}V_{us}^*)$ encode short distance information (masses of heavy particles, CKM factors), are perturbatively calculable, and are known at NLO in α_s [3]. The calculation of $\langle\pi\pi|Q_i(\mu)|K\rangle$ requires non-perturbative methods capable to keep track of scale and scheme dependence of local operators, as observables do not depend on these conventional choices.

In terms of Wilson coefficients and local operators ϵ'/ϵ has the following expression

$$\Big|\frac{\epsilon'}{\epsilon}\Big| = \mathrm{Im}(V_{td}V_{ts}^*) \cdot \Big[P^{(1/2)} - P^{(3/2)}\Big] \tag{1.8}$$

$$P^{(1/2)} = r\sum_i\ y_i(\mu)\frac{\mathrm{Disp}[\langle Q_i(\mu)\rangle_0]}{\cos\chi_0}(1-\Omega_{\mathrm{IB}}) \tag{1.9}$$

$$P^{(3/2)} = \frac{r}{\omega}\sum_i\ y_i(\mu)\frac{\mathrm{Disp}[\langle Q_i(\mu)\rangle_2]}{\cos\chi_2} \tag{1.10}$$

with $r = \dfrac{G_F\omega}{2|\epsilon|\mathrm{Re}A_0}$ and $\omega = \dfrac{\mathrm{Re}A_2}{\mathrm{Re}A_0}$. Modulo isospin-breaking corrections denoted by Ω_{IB}, $P^{(1/2)}$ and $P^{(3/2)}$ represent the contributions to ϵ'/ϵ due to $I=0$ and $I=2$ final states ($\langle Q_i\rangle_{0,2}$ denote matrix elements with pions in $I=0$ or $I=2$). The dominant contribution to $P^{(1/2)}$ comes from the gluonic penguin Q_6, while

$P^{(3/2)}$ is dominated by the electroweak penguin Q_8 [4]. In standard analyses one uses as input from phenomenology $\mathrm{Im}(V_{td}V_{ts}^*) = (1.31 \pm 0.1) \times 10^{-4}$, r, and ω, while theory input is $y_i(\mu)$, $\langle Q_6(\mu)\rangle_0$ and $\langle Q_8(\mu)\rangle_2$, as well as the isospin-breaking factor Ω_{IB}. Presently, the dominant uncertainties reside in $\langle Q_6(\mu)\rangle_0$, $\langle Q_8(\mu)\rangle_2$, and Ω_{IB}. For calculations based on lattice QCD I refer to the talk by L. Giusti [5], while here I focus on a selection of recent non-lattice results, with no attempt to cover all recent work in this field.

2 Matrix elements beyond factorization

In discussing $K \to \pi\pi$ matrix elements two organizing principles prove very useful:

1. *Chiral symmetry and Chiral Perturbation Theory* . This is relevant due to the nature of K and π, Goldstone modes associated with spontaneous breaking of chiral symmetry. As a consequence, matrix elements admit a low energy expansion in $(p/\Lambda_\chi)^{2n}, n = 0, 1, ...,$ where $p^2 \sim M_\pi^2, M_K^2$, while $\Lambda_\chi \sim 1$ GeV.
2. $1/N_c$ *expansion.* At large N_c four-quark operators factorize in the product of chiral currents and densities, with known hadronization. At LO in ChPT this implies:

$$\langle Q_6(\mu)\rangle_0^\infty = -4\sqrt{2}\,(F_K - F_\pi)\left(\frac{M_K^2}{m_s(\mu) + m_d(\mu)}\right)^2$$

$$\langle Q_8(\mu)\rangle_2^\infty = 2\,F_\pi\left(\frac{M_K^2}{m_s(\mu) + m_d(\mu)}\right)^2$$

Deviations from factorization are parameterized by the B_i factors, according to $\langle Q_i\rangle = \langle Q_i\rangle^\infty \cdot B_i$. I now discuss attempts to go both beyond factorization (at leading chiral order) and beyond leading chiral order.

Electroweak penguins

Chiral symmetry relates $K \to \pi\pi$ matrix elements of $Q_{7,8}$ to the correlator $\Pi_{LR}(q^2)$ [6,7,8], related to the transverse part of $\langle 0|T\left((\bar{d}_L\gamma^\mu u_L)\,(\bar{u}_R\gamma^\nu d_R)\right)|0\rangle$. At leading chiral order, using dimensional regularization and $\overline{\mathrm{MS}}$ subtraction, the operators Q_7 and Q_8 bosonize as follows:

$$Q_7(\mu) \to \hat{O}\,\left[\frac{3(d-1)\mu^{4-d}}{(4\pi)^{d/2}\Gamma(d/2)}\int_0^\infty dQ^2\,Q^d\,\Pi_{LR}(Q^2)\right]_{\overline{\mathrm{MS}}}$$

$$Q_8(\mu) \to \hat{O}\,\left[\frac{1}{C_{\mathrm{w}}(\mu)}\,\mu^6\,\Pi_{LR}(\mu^2)\;+\;O(1/\mu^2)\right]$$

in terms of the chiral operator $\hat{O} = \langle\lambda U Q U^\dagger\rangle$. In intuitive terms one can state that $\langle Q_7\rangle$ is given by the area under the curve in Fig. 1 after subtraction of the tail, which generates an UV divergence. On the other hand, $\langle Q_8\rangle$ governs the normalization of the divergent tail, modulo the calculable and scheme-dependent coefficient $C_{\mathrm{w}}(\mu)$. An OPE calculation of $\Pi_{LR}(Q^2)$ in the deep Euclidean region allows one to obtain C_{w}, and therefore to assign to both Q_7 and Q_8 the needed scale and scheme dependence. The

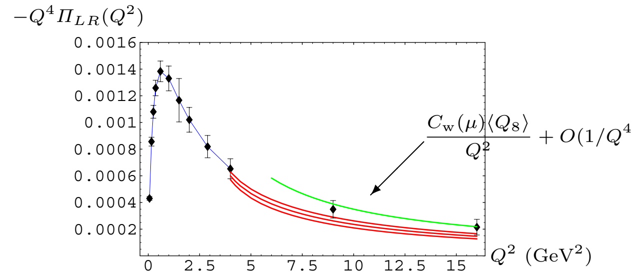

Fig. 1. Electroweak penguins in pictures

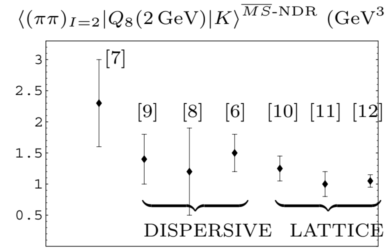

Fig. 2. Recent determinations of $\langle Q_8\rangle_2$

LR correlator is special because the corresponding spectral function $\rho(s) = \frac{1}{\pi}\mathrm{Im}\Pi_{LR}(s)$ is experimentally known from τ decays, and this in turn allows for a data-based evaluation of $\langle Q_{7,8}\rangle_{0,2}$ via dispersion relations. Theoretical uncertainties arise since the dispersive integrals extend from threshold to $s = \infty$, while data points stop at $s = m_\tau^2$. The numerical evaluation of [6] is based on complementing τ data with QCD chiral sum rules (RWM) as well as the use of finite-energy sum rules (FESR). RWM allows one to map out the correlator at various Q^2 with increasing uncertainties as Q^2 grows (see Fig. 1). On the other hand FESRs determine the coefficients of the asymptotic expansion at high Q^2. The resulting behavior is represented in Fig. 1 by the continuous curves starting at $Q^2 = 4\mathrm{GeV}^2$.

In Fig. 2 I report a compilation of results for $\langle Q_8\rangle_2$ from several recent calculations, using different techniques. The lattice data (extrapolated to chiral limit) are in the quenched approximation and quoted error is statistical only. Within errors, there is a reasonable agreement between lattice and dispersive results. The agreement persists after including estimates for NLO chiral corrections. For example, the results of [12] and [6] compare as follows:

$$\langle(\pi\pi)_{I=2}|Q_8(2\,\mathrm{GeV})|K\rangle^{\mathrm{LATT}} = (0.69 \pm 0.12)\ \mathrm{GeV}^3$$

$$\langle(\pi\pi)_{I=2}|Q_8(2\,\mathrm{GeV})|K\rangle^{\mathrm{DISP}} = (1.10 \pm 0.36)\ \mathrm{GeV}^3$$

Gluonic penguins

A very interesting recent calculation of Q_6 beyond factorization is reported in [13]. The authors show that, including subleading effects of order n_f/N_c, Q_6 hadronizes *at leading chiral order* ($O(p^2)$) as follows:

$$Q_6(\mu) \to \langle\lambda D_\mu U^\dagger D_\mu U\rangle\,\left\{\frac{-16L_5\langle\bar{q}q\rangle^2}{F^6} + \frac{8n_f\mu^{4-d}}{(4\pi)^{d/2}\Gamma(d/2)F^4}\int_0^\infty dQ^2\,Q^{d-2}\,\mathcal{W}(Q^2)\right\}_{\overline{\mathrm{MS}}}$$

where $\mathcal{W}(Q^2)$ is related to the transverse part of the 4-point function $\langle 0|T\left((\bar{s}_L q_R)(\bar{q}_L d_R)(\bar{d}_R\gamma^\mu u_R)\right.$ $\left.(\bar{u}_R\gamma^\nu s_R)\right)|0\rangle$ with soft current insertions. Model independent information on the correlator $\mathcal{W}(Q^2)$ is known only at low Q^2 (from CHPT) and high Q^2 (from the OPE). The authors construct a meromorphic interpolating form of $\mathcal{W}(Q^2)$ (as required by large N_c QCD) with only a finite number of pole singularities. The residues are fixed in order to match the model-independent long and short distance results. Numerically, large corrections to factorization are found, corresponding to $B_6(1\text{GeV}) \sim 3$, as well as a correlated enhancement of $\text{Re}A_0$, through $\langle Q_2\rangle_0$. In principle the method is improvable, by adding more constraints, and it would be very interesting to check the stability of the result against addition of more poles. Similar results have been found so far within ENJL model [14], but not within lattice calculations (where $B_6 < 1$).

At NLO in CHPT, one needs to include chiral loops with one insertion of Q_6 from order p^2, as well as local contributions induced by Q_6 at order p^4. The loops are known to be very important in the $I = 0$ channel due to strong rescattering, and provide a sizable enhancement ($\sim 40\%$) of $\langle Q_i\rangle_0$ [4,15]. The NLO local couplings are only known at leading order in $1/N_c$ (factorization) or within models. The induced uncertainty can be estimated by requiring that the size of $1/N_c$ corrections to the couplings be comparable to the size of their scale dependent component, suppressed in the $1/N_c$ counting [15].

Table 1. Summary table of isospin violation in ϵ'/ϵ . Each entry is in units of 10^{-2}. See text and [16]

	$\alpha = 0$		$\alpha \neq 0$	
	LO	LO+NLO	LO	LO+NLO
Ω_{IB}	11.7	15.9 ± 4.5	18.0 ± 6.5	22.7 ± 7.6
Δ_0	-0.004	-0.41 ± 0.05	8.7 ± 3.0	8.3 ± 3.6
$f_{5/2}$	0	0	0	8.3 ± 2.4
Ω_{eff}	11.7	16.3 ± 4.5	9.3 ± 5.8	6.0 ± 8.0

3 Isospin violation in ϵ'/ϵ

It is well known that isospin-breaking effects (of order α or $(m_u - m_d)/\Lambda_{QCD}$) can play an important role in the prediction of ϵ'/ϵ , as they are enhanced by the ratio $\frac{\text{Re}A_0}{\text{Re}A_2} \sim 22$. Historically such effects have been collected in the parameter Ω_{IB} appearing in (1.9), defined as

$$\Omega_{\text{IB}} = \frac{\text{Re}A_0}{\text{Re}A_2} \cdot \frac{\text{Im}A_2^{\text{non}-\text{Q}_8}}{\text{Im}A_0} . \tag{3.1}$$

A complete analysis of first order isospin breaking [16] reveals that other effects have to be included at the same order, and $\Omega_{\text{IB}} \to \Omega_{\text{eff}} = \Omega_{\text{IB}} - f_{5/2} - \Delta_0$. Here $f_{5/2}$ is the $\Delta I = 5/2$ contribution ($O(\alpha)$) to the overall normalization, while Δ_0 represents the isospin breaking effect in the amplitude A_0.

In [16] a complete calculation to NLO in CHPT is performed. The full loop corrections are included, while local couplings are estimated in leading $1/N_c$. Potentially large deviations from large N_c predictions for the LO couplings have been taken into account in assessing the errors. Results are reported in Table 1. It is interesting to note that the final figure for Ω_{eff} results from significant cancellations among competing effects.

4 Summary

It is not possible in such a limited space to report about all recent calculations of ϵ'/ϵ . A more complete list of references can be found in [17].

My personal view is that we have reached a reasonable agreement between lattice and dispersive calculations of Q_8, which translates into a vertical band in the $P^{(1/2)}$-$P^{(3/2)}$ plane (see Fig. 3). There is at the moment no consensus for the Q_6 contribution, which would determine the vertical coordinate in the plot. Requiring that the Standard Model reproduces the experimental result (diagonal band in the plot) implies the range $\langle Q_6(2\text{GeV})\rangle_0^{\text{NDR}} \sim (-0.75 \pm 0.15)\,\text{GeV}^3$, or $B_6^{(1/2),\text{NDR}}(2\,\text{GeV}) \sim 1.26 \pm 0.25$.

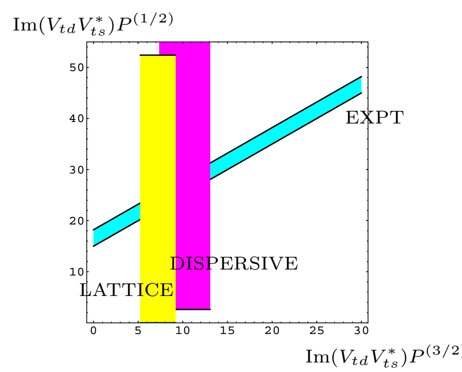

Fig. 3. Theory versus experiment

The above considerations lead to the simple message that ϵ'/ϵ is not (yet?) a quantitative test of the Standard Model, despite formidable theoretical efforts. However, given the considerable refinement of analytic and lattice techniques achieved in the past few years, the reduction of theoretical uncertainties to an acceptable level might be within reach.

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